Transfer Function Models and Sensitivity Analysis

Ajay M. Koshti, D. Sc. PE
NASA Johnson Space Center, Houston

SPIE Smart Structures/NDE 2023
Long Beach, CA, U. S. A.

Objective

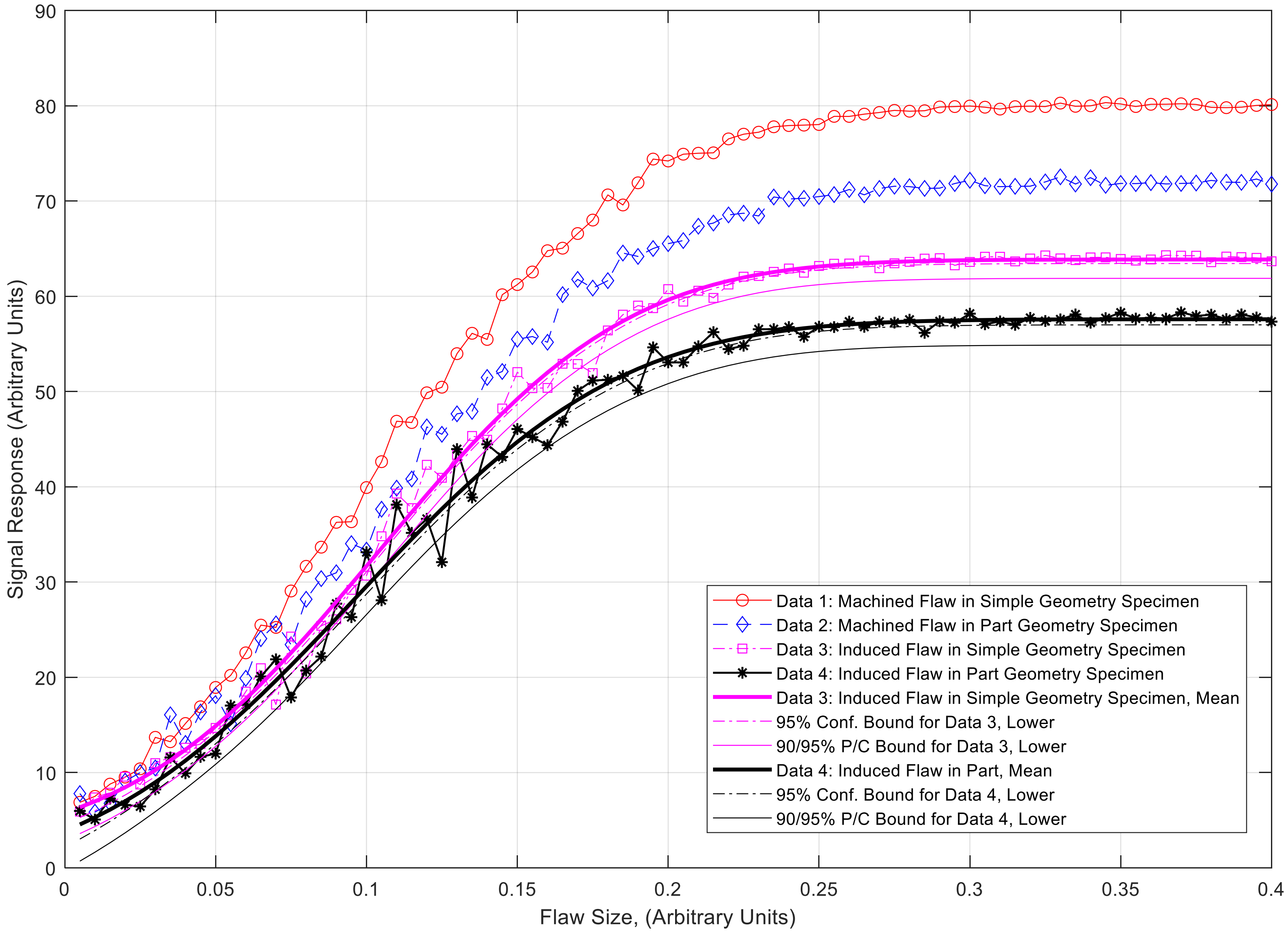
- Present a signal response transfer relationships model
- Present transfer function models
- Use simulated data to explain the models
  - Generate fictitious population data that follows transfer function assumptions
    - Assess selected sampling cases in determining flaw size estimate for
      - Forward case
        - Input flaw size: Target flaw size in part
        - Output flaw size: Mean predicted induced flaw size for demonstration in simple geometry specimen
      - Inverse case
        - Input flaw size: Mean induced flaw size for demonstration in simple geometry specimen
        - Output flaw size: Predicated 90/95 Target flaw size in part
  - Signal response data is chosen to have saturation limit
      - The upper saturation limits are different for differing data sets
      - The data is linear in middle signal response section and non-linear in upper and lower signal response knee regions
      - Saturating data characteristics is chosen as a more challenging but practical case of data encountered in NDE
- Demonstrate use of simulation to assist in transfer function data sampling parameters (flaw sizes and replicates)

Data Sets

		Flaw Type	
		Artificial or Machined Flaw (e.g., EDM, subscript e)	Induced Flaw (e.g., crack, subscript c)
Specimen Geometry	Simple Geometry (subscript 1)	Data 1: y_{e1}	Data 3: $y_{c,1}$
	Part Configuration or real part (subscript 2)	Data 2: $y_{e,2}$	Data 4: $y_{c,2}$

Note that subscript “1” is used for simple geometry specimens and subscript “2” is used for part or part configuration specimens. An example of simulated data for signal response transfer relationships model is provided below.

Example of Data with Known Characteristics



Signal response transfer relationships model inputs and their relationships

Row No.	Description of Input	Notation	Example Values	Comments
1	Amplitude (% full screen height or % FSH)	$\bar{y}_{e1,max}$	80	Sets saturation limit for Data 1.
2	Noise, standard deviation (% FSH)	σ_1	0.2	For simple geometry, subscript 1 is used and for part, subscript 2 is used.
3	Noise ratio, part-to-simple geometry (% FSH)	$R = \frac{\sigma_2}{\sigma_1}$ (1A)	1.5	$\sigma_2 = R\sigma_1$ (1B)
4	EDM size variation equivalent to signal response standard deviation (% FSH)	σ_{e1}^{x}	0.03	
5	EDM flaw size standard deviation ratio, part-to-simple geometry (% FSH)	$f_A = \frac{\sigma_{e2}^{x}}{\sigma_{e1}^{\Delta x}}$ (2A)	1.5	$\sigma_{e2}^{x} = f_A \sigma_{e1}^{x}$ (2B)
6	Induced flaw signal standard deviation ratio, part-to-simple geometry (% FSH)	$f_B = \frac{\sigma_{c1}^{x}}{\sigma_{e1}^{x}}$ (3A)	2.0	$\sigma_{c1}^{x} = f_B \sigma_{e1}^{x}$ (3B) $\sigma_{c2}^{x} = f_A f_B \sigma_{e1}^{x}$ (4)
7	Mean signal response ratio, part-to-simple geometry (% FSH)	$R_A = \frac{\bar{y}_{e2,max}}{\bar{y}_{e1,max}}$ (5)	0.9	
8	Mean signal response ratio, Crack-to-EDM (% FSH)	$R_B = \frac{\bar{y}_{c1,max}}{\bar{y}_{e1,max}}$ (6)	0.8	$\bar{y}_{c2,max} = R_A R_B \bar{y}_{e1,max}$ (7)
9	Minimum flaw size (arbitrary units)	$x_{c2,min}$	0.005	Also applicable for Data 1, Data 2, Data 3, and Data 4.
10	Maximum flaw size (arbitrary units)	$x_{c2,max}$	0.4	Also applicable for Data 1, Data 2, Data 3, and Data 4.
11	Flaw size spacing (arbitrary units)	$\Delta_{e1}^{x} = \Delta_{e2}^{x} = \Delta_{c1}^{x} = \Delta_{c2}^{x}$ (8)	0.005	
12	Flaw size for 50% of saturated signal response function (arbitrary units)	$x_{e1}^{50} = x_{e2}^{50} = x_{c1}^{50} = x_{c2}^{50}$ (9)	0.1	$x_{norm} = \frac{(x - x_{e1}^{50})}{\sigma_{e1}^{x}}$ (10)
13	Standard deviation of saturated signal response function (% FSH)	$\sigma_{e1}^{x} = \sigma_{e2}^{x} = \sigma_{c1}^{x} = \sigma_{c2}^{x}$ (11)	0.1	

Signal response transfer relationships model

$$y_{e1} = \frac{\bar{y}_{e1,max}}{2}(1 + erf(x_{norm} + randn(\sigma_{e1}^{x}))) + randn(\sigma_1)$$

$$y_{e2} = \frac{\bar{y}_{e2,max}}{2}(1 + erf(x_{norm} + randn(\sigma_{e2}^{x}))) + randn(\sigma_2),$$

$$y_{c1} = \frac{\bar{y}_{c1,max}}{2}(1 + erf(x_{norm} + randn(\sigma_{c1}^{x}))) + randn(\sigma_1),$$

$$y_{c2} = \frac{\bar{y}_{c2,max}}{2}(1 + erf(x_{norm} + randn(\sigma_{c2}^{x}))) + randn(\sigma_2)$$

$$\bar{y}_{c2} = \frac{\bar{y}_{c1}\bar{y}_{e2}}{\bar{y}_{e1}}$$

$$\sigma_{c2}^{x} = \frac{\sigma_{c1}^{x}\sigma_{e2}^{x}}{\sigma_{e1}^{x}}$$

where,

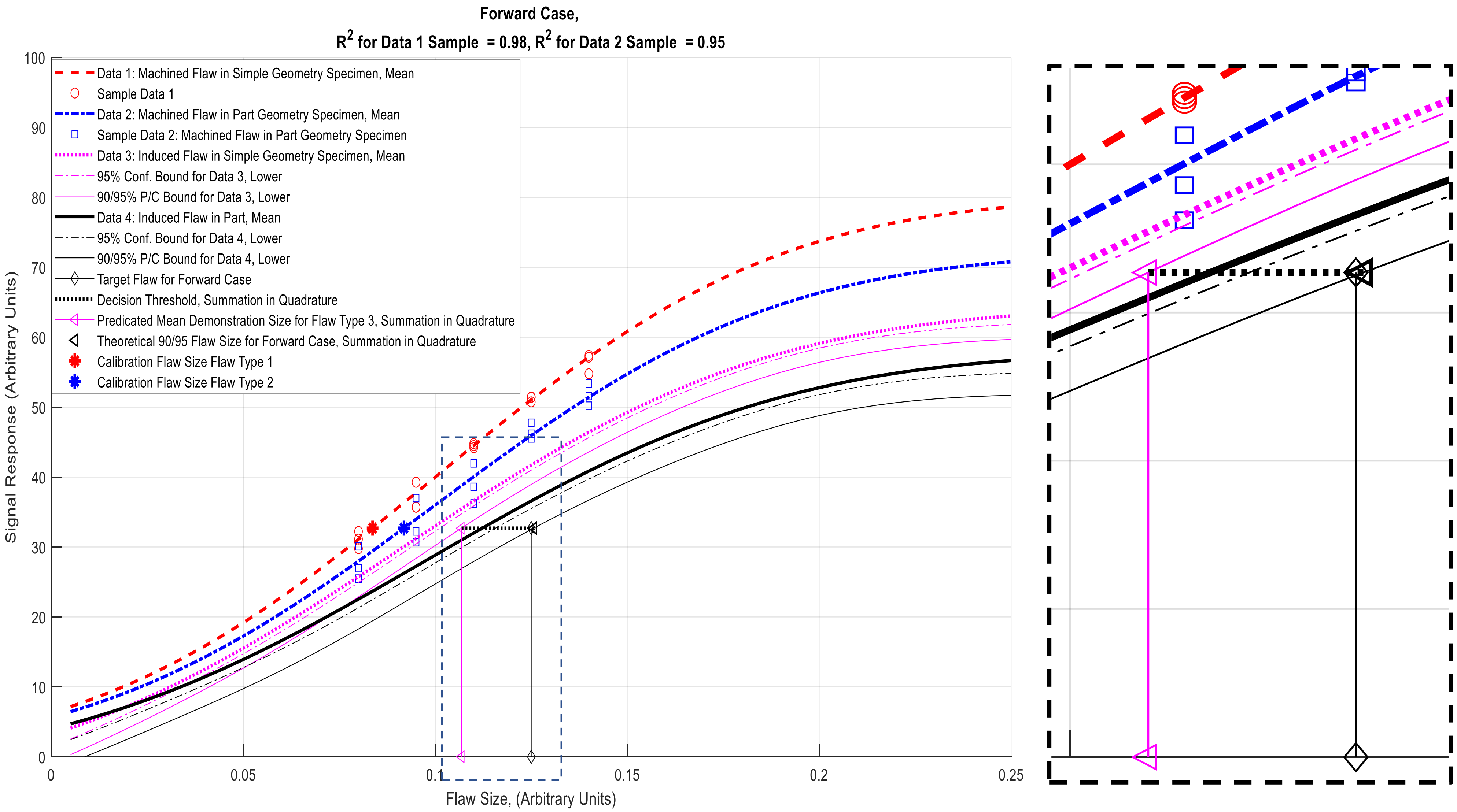
erf = error function, and

randn = random number generator function with Gaussian distribution.

x = flaw size.

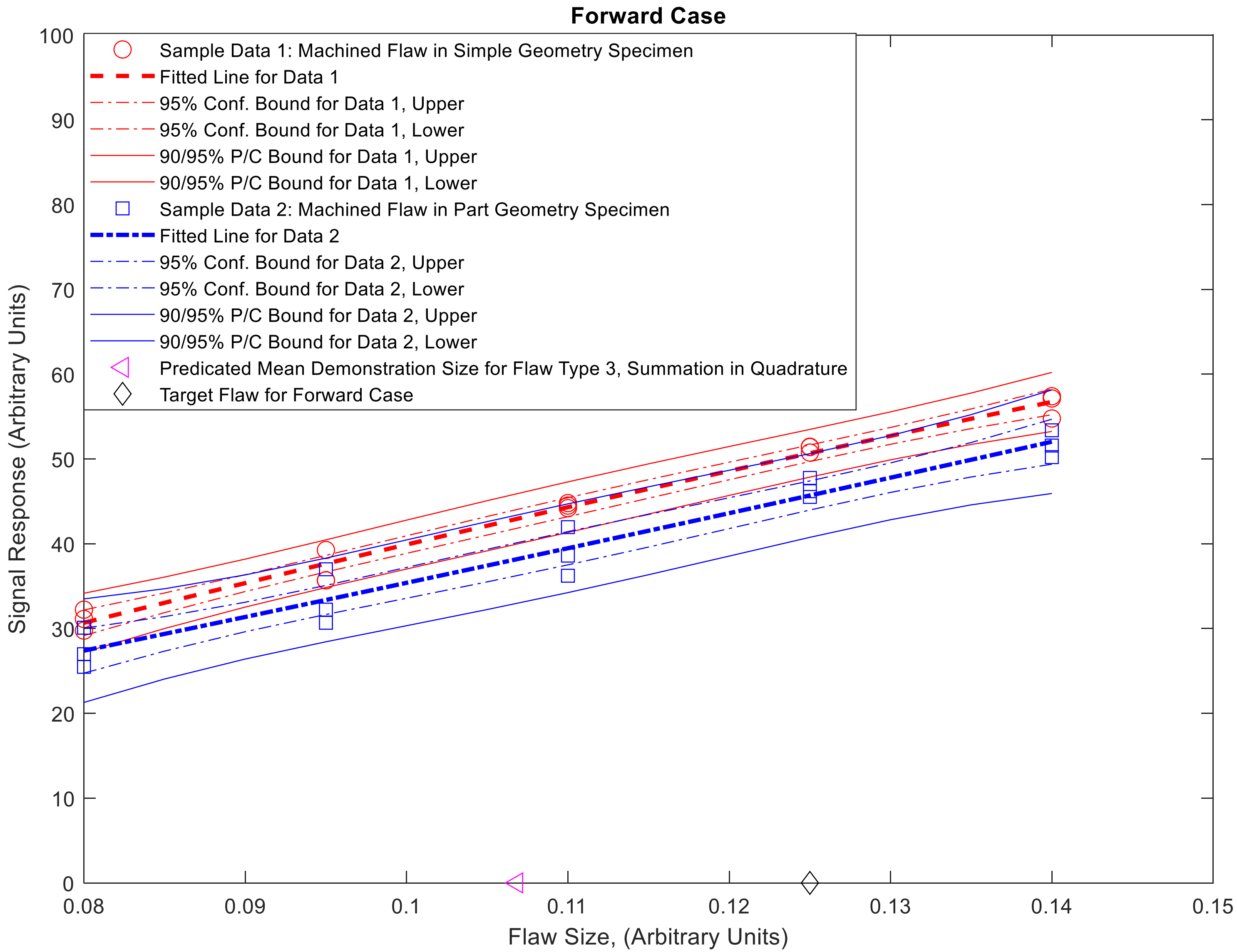
Forward Case: Sample Data 1 and Data 2 overlaid on the respective mean curves and analysis

FORWARD CASE



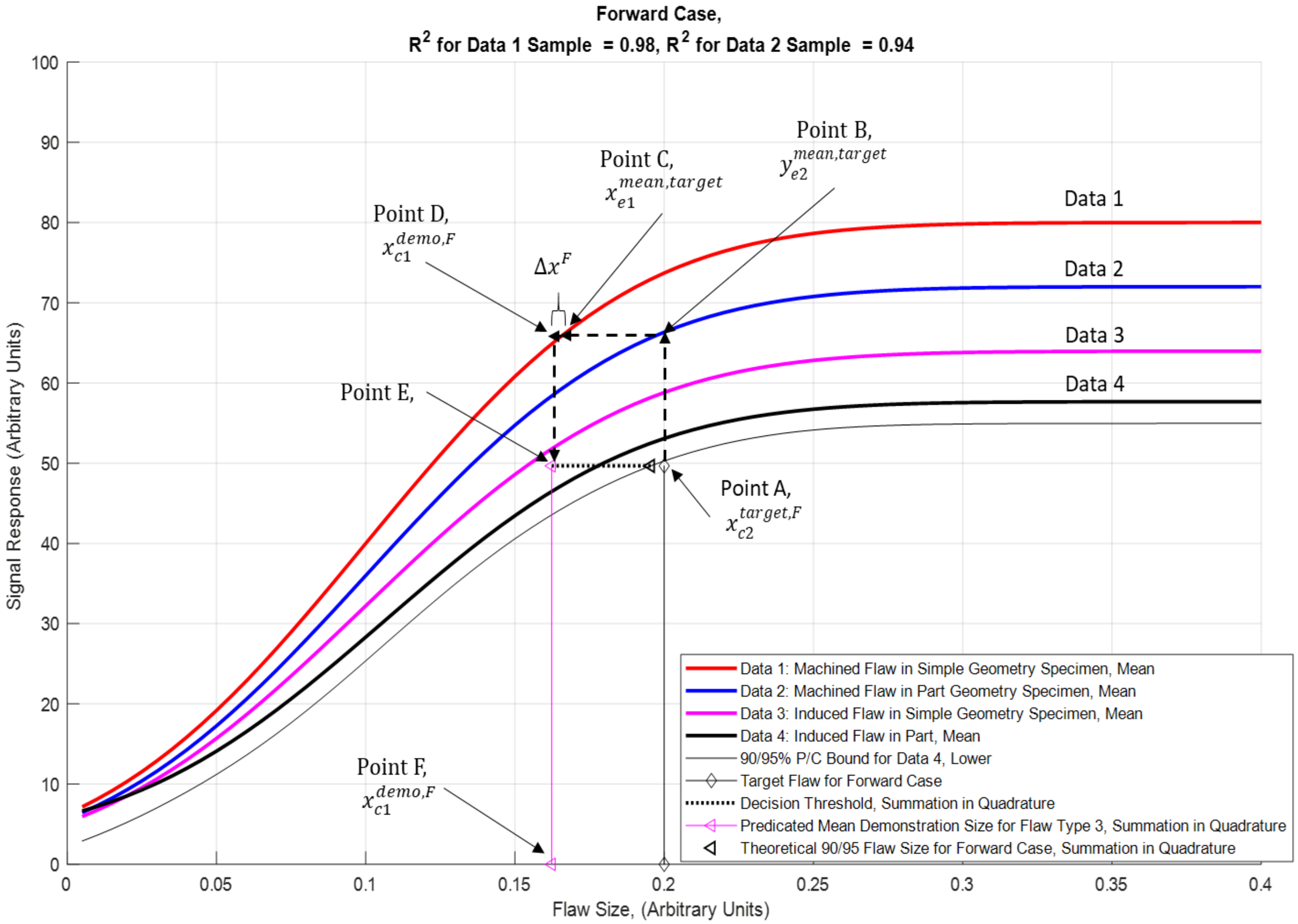
Forward Case: Fitted second order polynomials through sample Data 1 and Data 2

FORWARD CASE



Ajay M. Koshti, SPIE Smart Structures/NDE 2023, Long Beach, CA

Forward Case

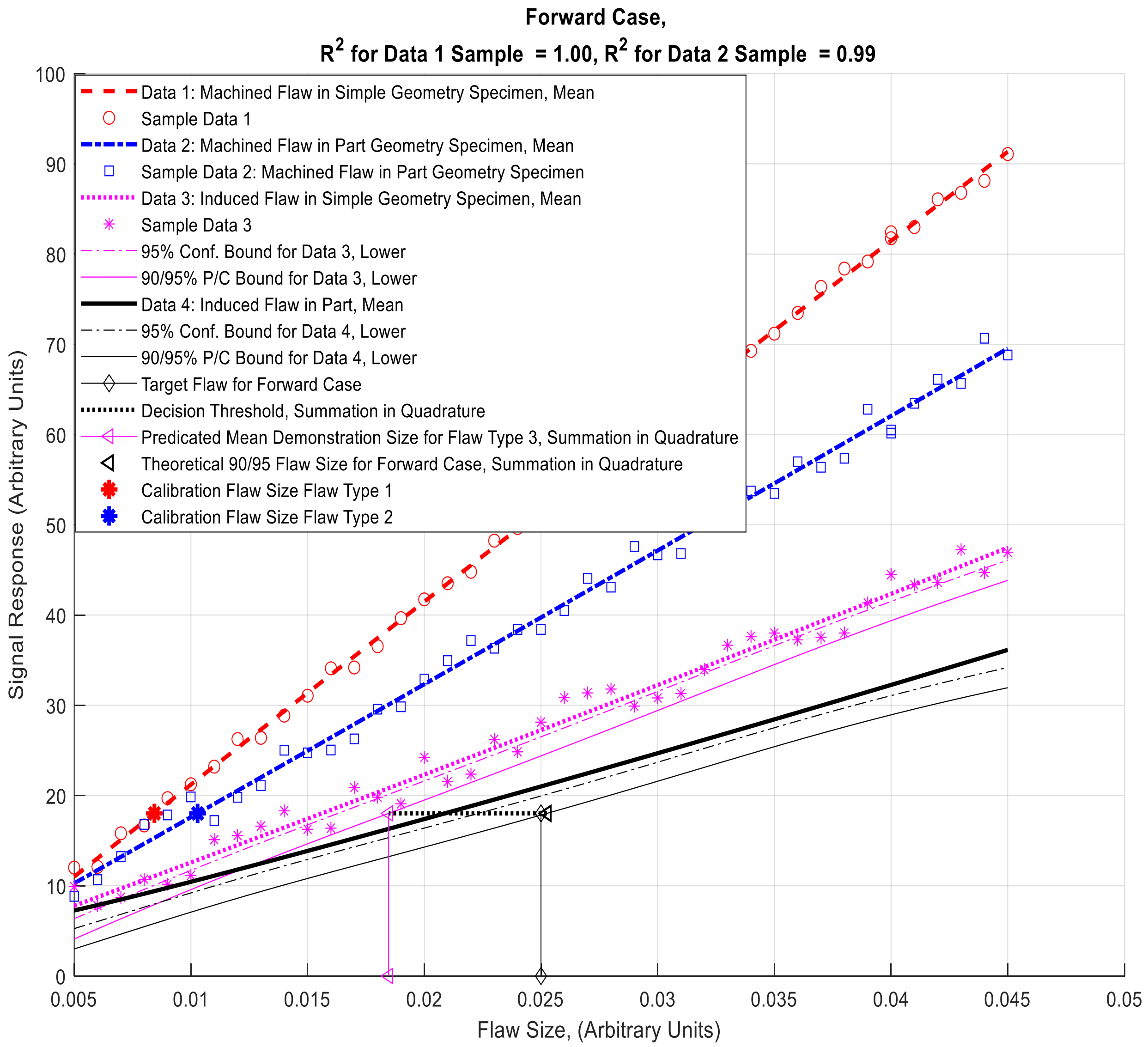


$$x_{c1}^{demo,F} = x_{e1}^{mean,target} - \Delta x^F$$

$$\Delta x^F = \sqrt{\left(\Delta x_{e1}^{target,F}\right)^2 + \left(\Delta x_{e2}^{target,F}\right)^2 + \left(\Delta x_{c1}^{target,F}\right)^2}$$

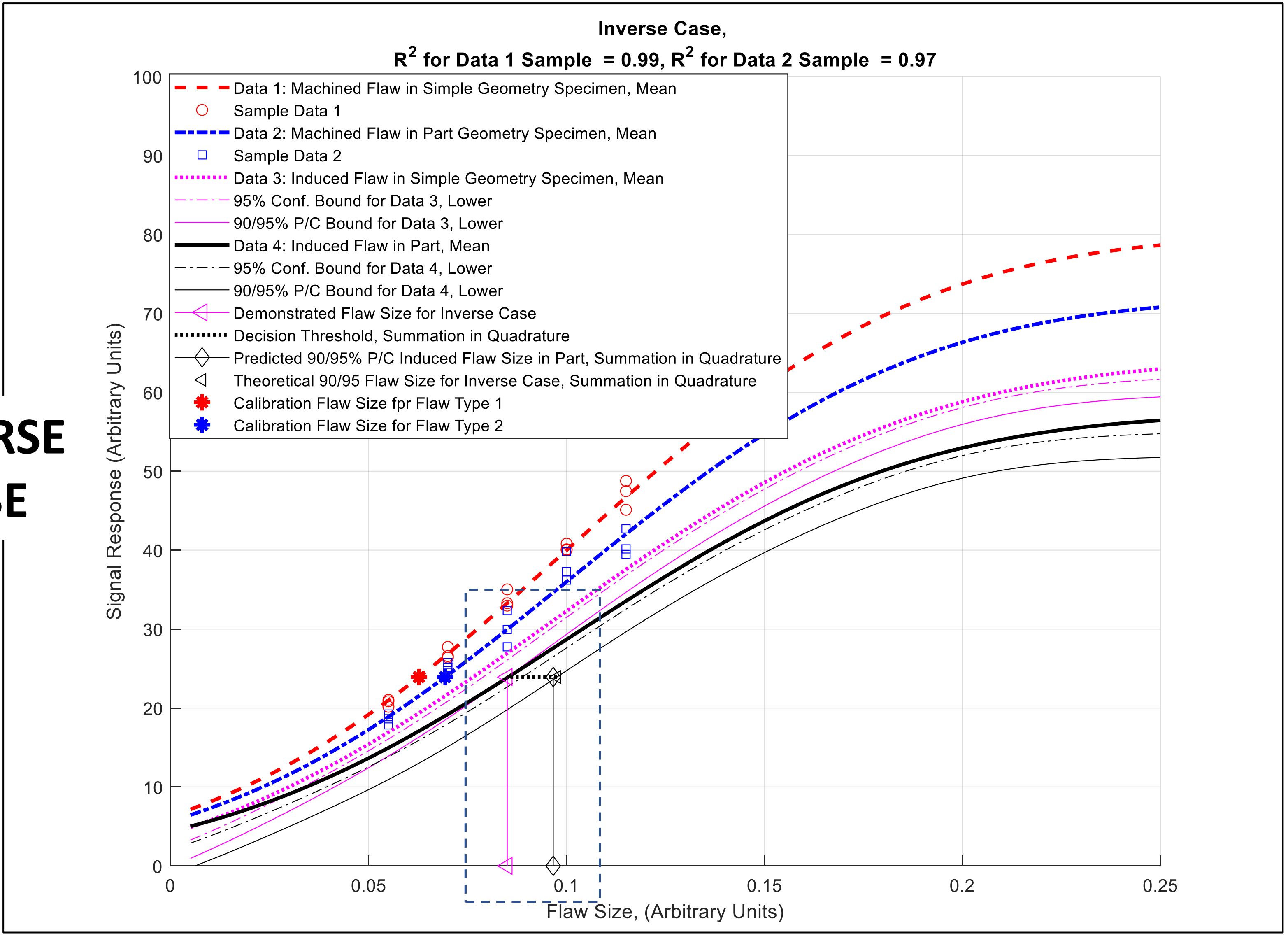
Point A ---> point B ---> point C ---> point D ---> point E ---> point F

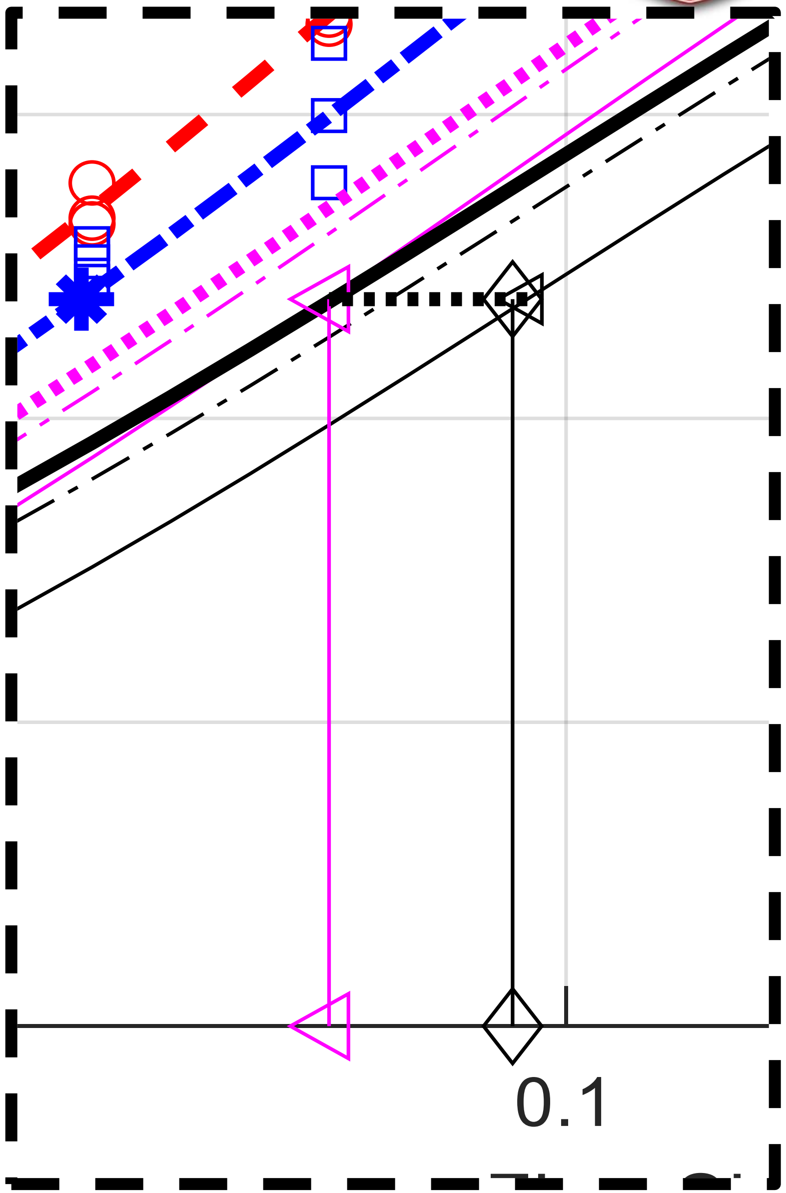
Example of forward case using simulated data



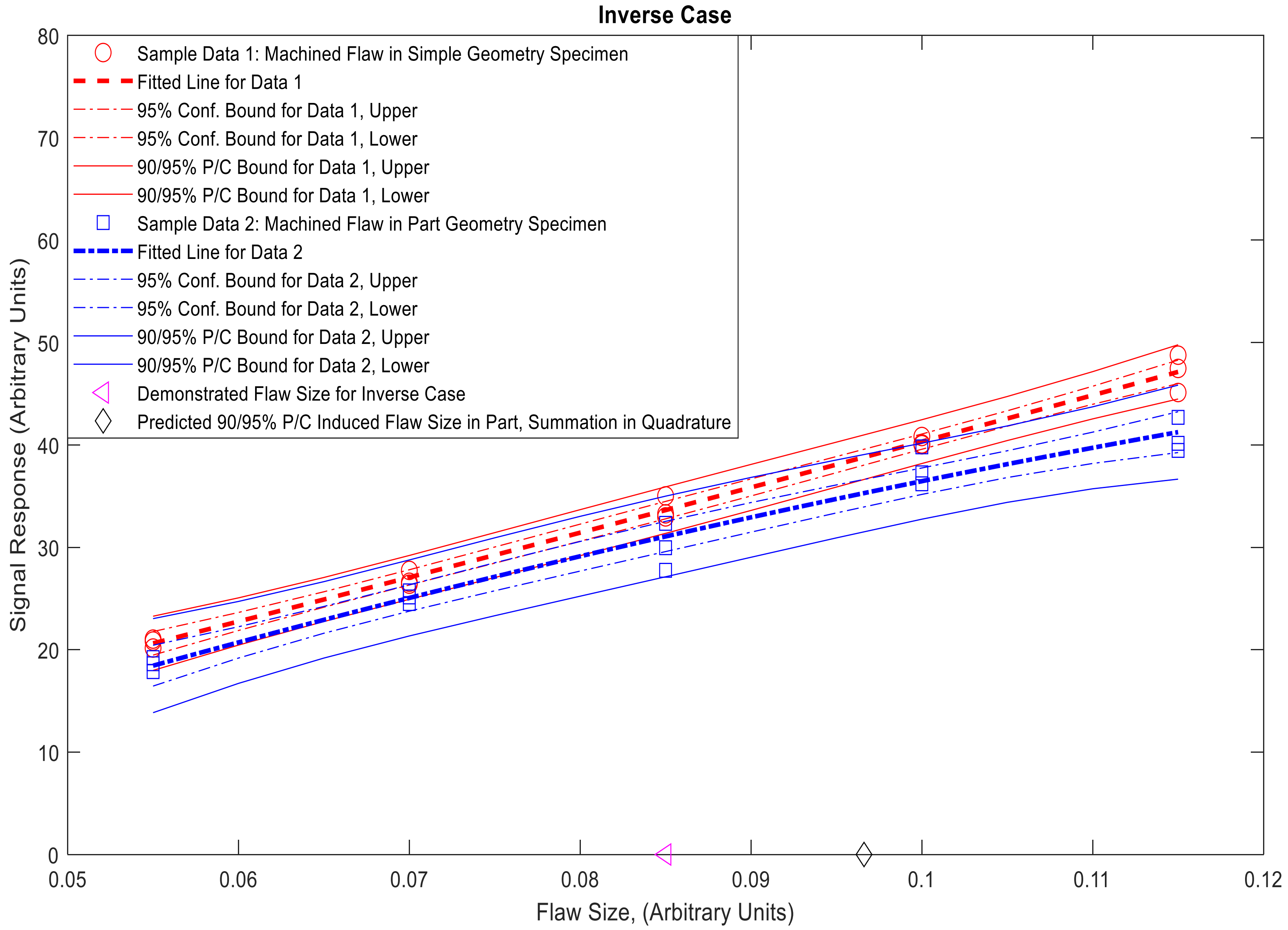
Sample Data 1 and Data 2 overlaid on the respective mean curves and analysis for inverse case

INVERSE CASE





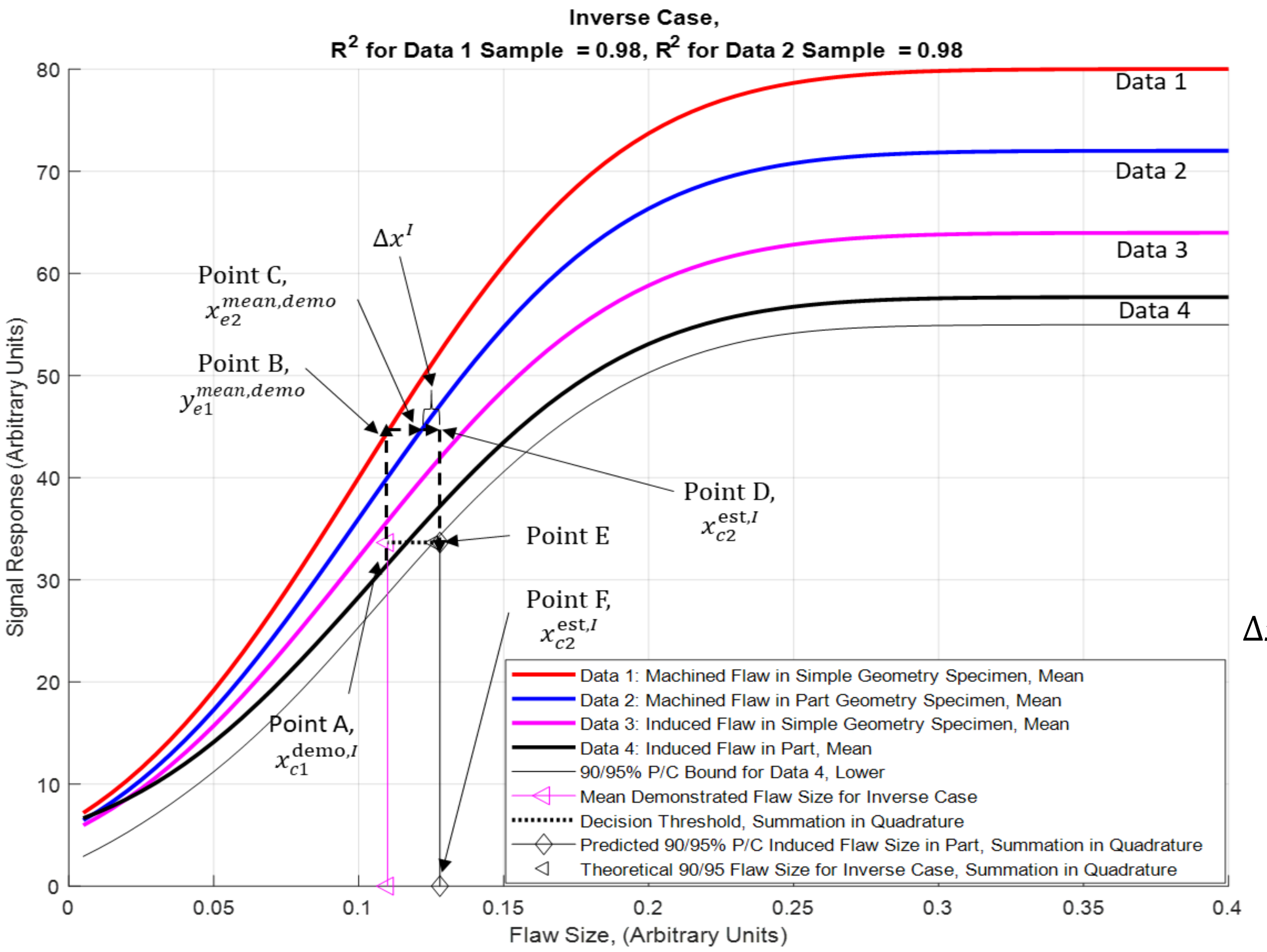
Fitted second order polynomials through sample Data 1 and Data 2



INVERSE CASE

Linear or quadratic fit

Inverse Case

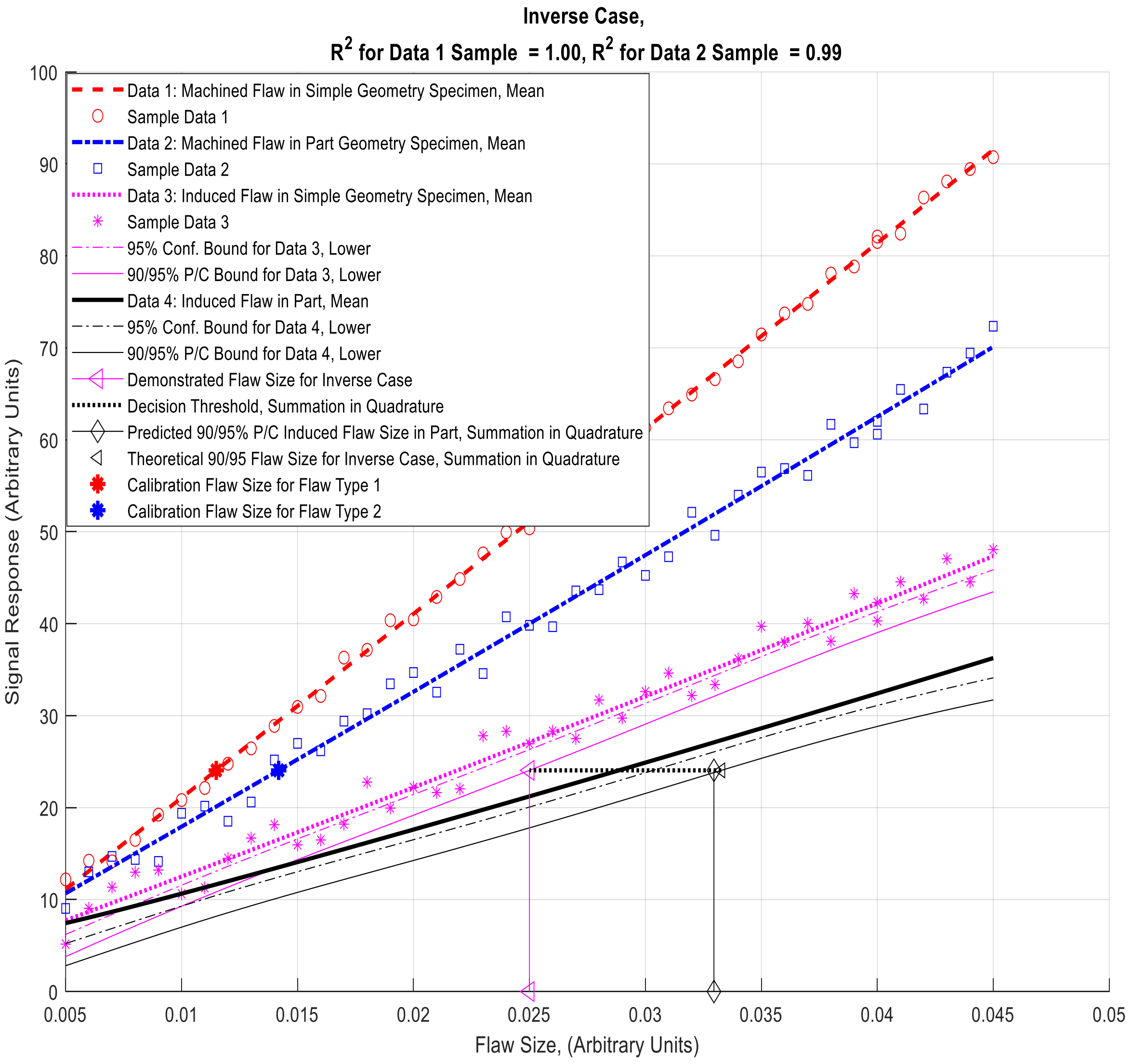


$$x_{c2,90/95}^{est} = x_{e2}^{mean,demo} + \Delta x^{I}$$

$$\Delta x^{I} = \sqrt{\left(\Delta x_{e1}^{demo,I}\right)^2 + \left(\Delta x_{e2}^{demo,I}\right)^2 + \left(\Delta x_{c1}^{demo,I}\right)^2}$$

Point A ---> point B ---> point C ---> point D ---> point E ---> point F

Example of inverse case using simulated data



Ajay M. Koshti, SPIE Smart Structures/NDE 2023, Long Beach, CA

Inferences

- For good Transfer Function analysis results
  - Predicted flaw size should be Interpolated
- Some sampling cases with small sample size are considered
  - Set 1: 4 sizes x 2 replicates, Set 2: 4 sizes x 2 replicates
  - Set 1: 5 sizes x 2 replicates, Set 2: 5 sizes x 2 replicates
  - Set 1: 3 sizes x 3 replicates, Set 2: 3 sizes x 3 replicates
- Quadratic or (2nd order polynomial) fit is considered for data range (which does not have an inflection point)
- Data should be interpolated
- Simulated data can be used to assess different sampling cases to choose optimal sampling
- Simulation provides a method to assess whether transfer function is needed
  - If absolute (Input flaw size – Output flaw size) > a set value, then transfer function is needed.

Conclusions

- The signal response transfer relationships model is devised first.
- Assuming the signal response transfer relationships model is applicable to the NDE application, transfer function forward or inverse methods have been devised with an objective to provide adequate confidence to the assumed signal response transfer relationships model.
- For non-linear data, transfer function simulation is advised to assess applicability of transfer function method.
- The transfer function approach is a risk assessment approach as 90/95% POD/confidence cannot be demonstrated directly.
- The signal response transfer relationships model needs to be validated with empirical data and then transfer function model needs to be validated for desired POD and confidence on case-by-case basis.